Equivalence of Risk: A Mathematical Approach

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Abstract

Naval engineers express risk in terms of probability, severity, and risk acceptance authority. Currently, Naval Sea Systems Command Instruction (NAVSEAINST) 5000.8 specifies the typical format for reporting risk probability, severity, and acceptance authority. NAVSEAINST 5100.12 provides a chart whereby given a probability and severity, the risk can be found.

Although risk can be quantified, the human perception of risk often influences how risk is addressed. Risk perception does not necessarily map directly to probability and severity in a linear fashion. This paper proposes a Composite Risk Index (CRI) based on probability and severity that takes into account the human perception of equivalent risk. This paper also answers questions as to why the NAVSEAINST 5100.12 risk chart is logarithmic with respect to probability and proposes changes supporting the next NAVSEAINST 5100.12 revision.

Background

Risk is the potential for mishaps or other adverse variation in the cost, schedule, or performance of a program or its products (ref. 1). Safety risk is the potential for mishaps that result in injury, illness, death, equipment damage or loss, or environmental damage. All safety programs desire accurate risk quantification in order to provide a meaningful expression of risk. One factor complicating risk quantification is never having one single risk associated with a program. Safety engineers define a program's multitude of risks in terms of probability and severity for acceptance by the appropriate authority. To accept the total safety risk of a system, these authorities typically desire a composite risk estimate. Current methods of obtaining this composite risk estimate use summing techniques to add the individual risks associated with the system and produce a single number (ref. 2). This method seems intuitive; however, it is often difficult to determine particular probabilities or quantify severity. Accepted methods of quantifying severity include dollar amounts or severity of injury (ref. 3). Expressing severity in terms of cost establishes consistency, but it is still difficult to put a dollar amount on human injuries. For example, how many broken backs are equivalent to death? Furthermore, perception of what constitutes "high" risk may vary from project to project.

Discussion

This paper discloses methods to address these issues. Beginning with an analysis of the typical risk acceptance chart, let us investigate how the various Mishap Risk Indices (MRIs) relate to one another.

Figure 1 shows the typical MRI chart. This chart helps evaluate the level of risk as "high," "serious," "medium," or "low" based on probability and severity. The authority qualified to accept the safety risk increases from the Program Manager (PM), to the Program Executive Office (PEO), to the Component Acquisition Executive as the level of risk increases.

In Figure 1 severity is categorized in general terms of loss. This type of chart assigns a higher priority (lower number) to the more severe incidents. There is not a clear mapping of one severity to another in Figure 1. This paper proposes an approach to relate the severity categories to one another; whereby, there would be a relationship between the different severity categories.

MISHAP PROBABILITY LEVEL	MISHAP SEVERITY CATEGORY			
	I CATASTROPHIC	II CRITICAL	III MARGINAL	IV NEGLIGIBLE
A - FREQUENT	1A	2A	3A	4A
B - PROBABLE	1B	2B	3B	4B
C - OCCASIONAL	1C	2C	3C	4C
D - REMOTE	1D	2D	3D	4D
E - IMPROBABLE	1E	2E	3E	4E

1A-C, 2A-B	High Risk, CAE approval required
1D, 2C, 3A-B	Serious Risk, PEO approval required
1E, 2D-E, 3C-E, 4A-B	Medium Risk, PM approval required
4C-E	Low Risk, PM approval required

MISHAP SEVERITY

DESCRIPTION	CATEGORY	DEFINITION
Catastrophic	I	Death,permanent total disability, major ship/equipment damage or irreversible severe environmental damage.
Critical	II	Severe injury, permanent partial disability, system loss or damage, reversible environmental damage.
Marginal	III	Minor injury, minor equipment damage, or mitigatible environmental damage.
Negligible	IV	Less than minor injury, less than minor equipment damage or minor environmental damage.

MISHAP PROBABILITY

DESCRIPTION	LEVEL	INDIVIDUAL ITEM OR SYSTEM	ENTIRE FLEET
Frequent $P>=10^{-1}$	A	Likely to occur frequently	Continuously experienced.
Probable $10^{-1}>P>=10^{-2}$	B	Will occur several times in the life of the item.	Will occur frequently.
Occasional $10^{-2}>P>=10^{-3}$	C	Likely to occur some time in the life of an item.	Will occur several times.
Remote $10^{-3}>P>=10^{-6}$	D	Unlikely but possible to occur in the life of an item.	Unlikely, but can reasonable be expected to occur.
Improbable $10^{-6}>P$	E	So unlikely, it can be assumed occurrence may not be experienced.	Unlikely to occur, but possible.

Figure 1 – NAVSEAINST 5100.12 Risk Chart

It may not be obvious, but the MRI table is actually a three dimensional chart, mapping risk in terms of probability, severity, and level or acceptance authority. Figure 2 emphasizes this by depicting the risk level as height, the more risk, the higher the bar.

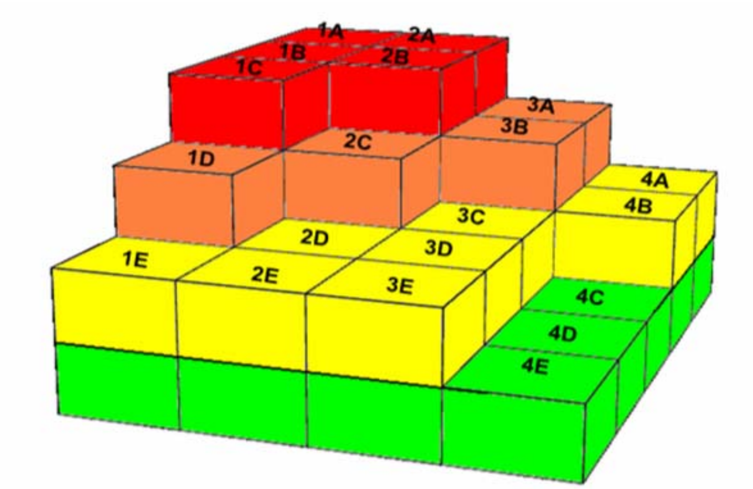

Figure 2 – MRI Chart in Three Dimensions

This paper starts with the premise that the NAVSEAINST 5100.12 chart accurately describes risk. From Figure 1, the probabilities are logarithmic, decreasing by 1 or more powers of 10 as the probability goes from “A” to “E.” A logarithmic chart may be appropriate. Some human perceptions are logarithmic in nature; examples include perceived brightness (apparent magnitude) and perceived loudness (decibel scale).

Risk perception is difficult to quantify and implicitly subjective. If left unchecked, it can lead to a difference between scientific facts and an exaggerated public reception of the dangers (ref. 4). Humans tend to perceive more risk from a single, dramatic incident than from constant, consistent, or less dramatic problems even if these cause more injuries or loss (ref. 4). Mishaps with lower probability are more noticeable on a logarithmic scale.

<u>Equivalence of Probability</u>

We start with the probability scale NAVSEAINST 5100.12 provides. To quantify severity, instead of guessing how to define severity in terms of cost or injury, we assign an index. High severity is the range between 0 and 1, Serious is from 1 to 2, and so forth. Each range has an expectation value associated with it. To determine the expectation value for a "frequent" probability, for example, the following calculations are performed:

$$\mu_1 = 10^{\frac{\int_0^1 x10^{-x}\,dx}{\int_0^1 10^{-x}\,dx}} = 10^{-0.323} = 0.475 \tag{1}$$

Figure 3 shows the expected range for each of the five probability ranges.

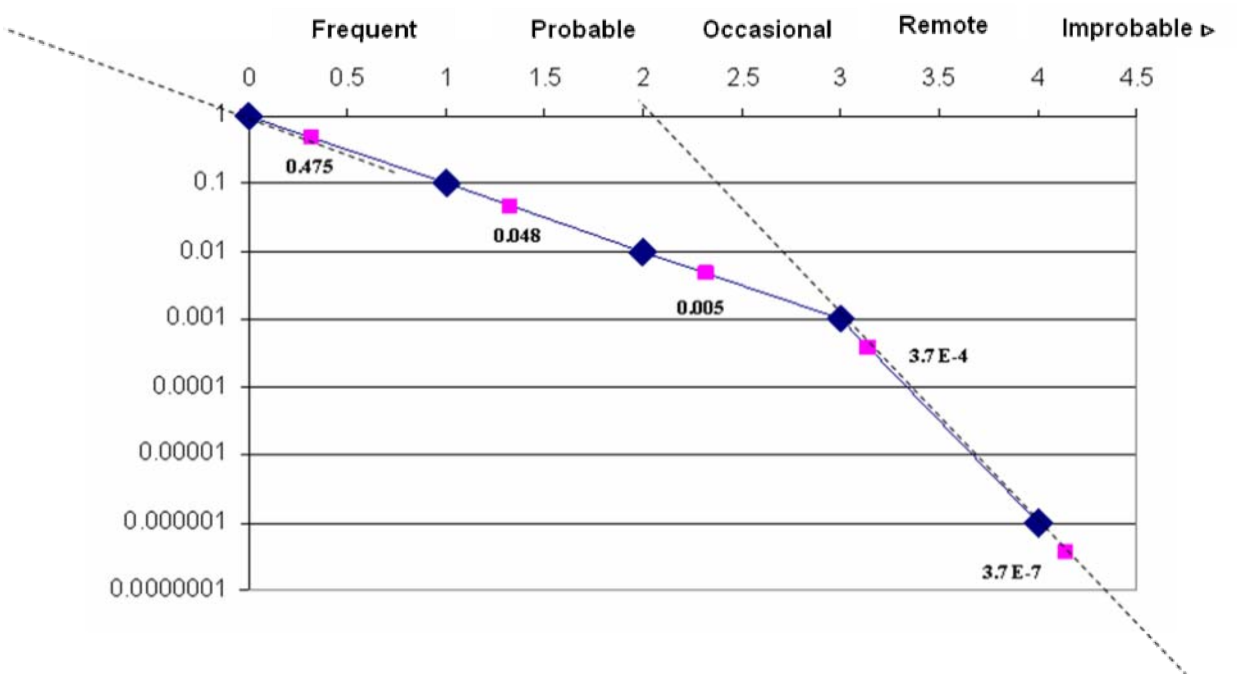

Figure 3 – Expectation Values for Probability Ranges

After the expectation value is determined, we can express one probability in terms of any other. For example, in order to determine how many "probable" hazards one would need in order to have the same chance as a frequent hazard, the following steps are used:

$$
\begin{aligned}
\mu_1 &= 1 - (1 - \mu_2)^n \\
1 - \mu_1 &= (1 - \mu_2)^n \\
\ln(1 - \mu_1) &= \ln(1 - \mu_2)^n \\
\ln(1 - \mu_1) &= n \ln(1 - \mu_2) \\
n &= \frac{\ln(1 - \mu_1)}{\ln(1 - \mu_2)} \\
n &= \frac{\ln(1 - 0.475)}{\ln(1 - 0.048)} = 13.24
\end{aligned}
\tag{2}
$$

One can see that it takes 13.24 "probable" hazards to have the same probability as a "frequent" hazard. This technique can be applied to all of the probability categories to obtain the results in Table 1.

Table 1 – Equivalence of Probabilities

Probable Hazards per Frequent Hazard	13.24
Occasional Hazards per Probable Hazard	10.22
Remote Hazards per Occasional Hazard	12.85
Improbable Hazards per Remote Hazard	1007.12

It is clear that any probability of any hazard can be expressed in terms of either multiple lesser probabilities or a portion of a greater probability.

Equivalence of Severity

Equivalence of Severity is much less straight forward. Previous approaches typically assign dollar values or give examples of various injuries in order to express the severity.

This approach uses perception. Although human perception of risk may be faulty, and should not be used as a sole determination of risk, it can be used as a starting point. We will assume the NAVSEAINST 5100.12 chart is "correct" in how the risks are labeled as "high," "serious," "medium," and "low." One thing to note is that for low and medium hazards, the acceptance authority is the same (PM). Therefore, the perception of risk may not be that great between low and medium (but there is still a difference). When a hazard is identified as being serious, risk can no longer be accepted at the Program Manager's level, but must be accepted by the PEO. The risk must be perceived as being higher, enough to warrant contacting an admiral. From serious to high risk, an even higher authority must accept risk.

For purposes of this analysis, we will examine one of the areas where perception of risk changes. Figure 4 shows the NAVSEAINST 5100.12 chart redrawn. Both probability and severity are preserved (marked by the number and letter in each square). Note that the vertical axis no longer measures probability when the chart is drawn in this manner.

It will be noticed that there is one line at the threshold between medium and serious that transgresses all four severity levels. Note that there is no transition to serious in the "negligible" category because the probability cannot exceed 100%.

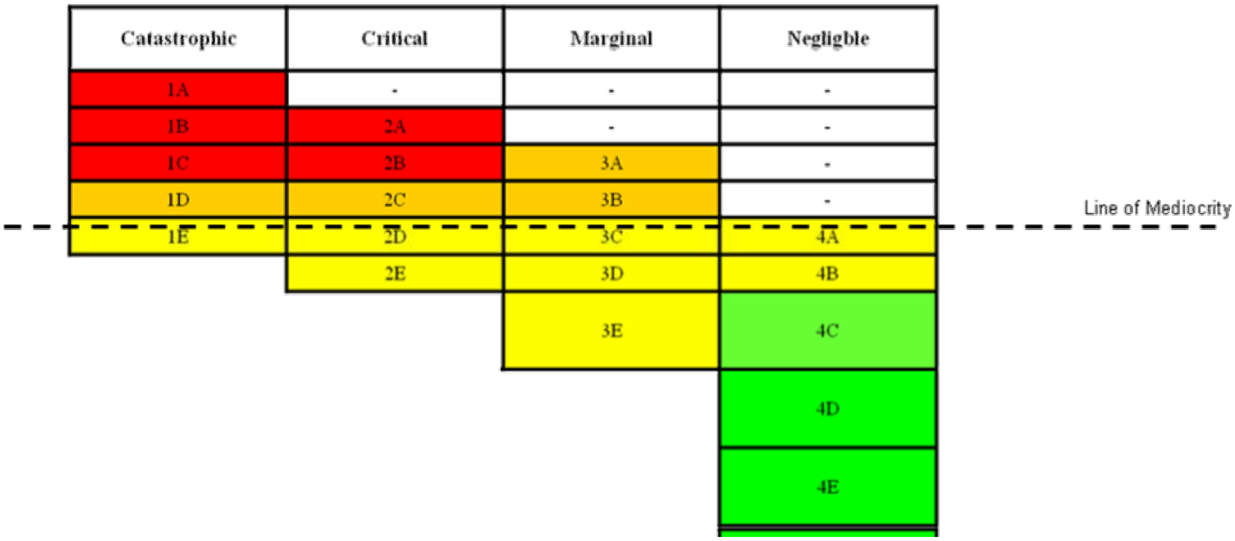

Figure 4 – NAVSEAINST 5100.12 Risk Chart Redrawn

The line between medium and serious (e.g., “Line of Mediocrity”) delineates what is serious from what is not. Any higher risk is perceived as being “more,” as we must act differently in order to obtain risk acceptance. This “call the admiral” approach can be used to mark a point.

The fundamental assumption is that at this point, risk is equivalent. That is, we perceive the risks of 1E, 2D, 3C, and 4A to be the same. Note that there is no assertion that risk is equivalent for any other horizontal line on the graph.

To equate severity, the approach is to take any two indices that have the same probability; for example, if we were to determine the relative difference of “marginal” to “negligible,” we could examine the indices 4A (negligible) and 3A (marginal). The only difference is the severity. Using the assumption that the risk is the same on the Line of Mediocrity, we see that a 4A is the same risk as a 3C. The 3C differs only from the 3A in probability. From Table 1, multiplying the 10.22 (occasional hazards per probable hazard) by the 13.24 (probable hazards per frequent hazard), we see that the 3A is equivalent to 135.31 3C hazards, and therefore equivalent to 135.31 hazards of the 4A category. We can therefore deduce that marginal is 135.31 times worse than negligible. Note that had we chosen 4B and 3B, for example, the resulting factor would still have been 135.31. In this manner, we can express severity in terms of other severities. Table 2 provides the equivalency of severities.

Table 2 – Equivalence of Severity

Critical Severities per Catastrophic Severity	1007.12
Marginal Severities per Critical Severity	12.85
Negligible Severities per Marginal Severity	135.31

Note that there is no such blanket statement for equating the actual risk. From our example, we can clearly see that the transition from medium to serious varies depending on the severity of the hazard, being 10.22, 12.85, and 1007.13 depending on whether the hazard is marginal, critical, or catastrophic, respectively. At first, this may seem counter-intuitive; however, it is a consequence of the way the risk chart was created and gives us an understanding on the effect probability has upon risk.

Summing of Risk: CRI

Up until this point, we have examined the risk of a hazard relative to other hazards, but have not assigned a specific value. It will be appreciated that what constitutes a “serious” or “medium” risk will be from the perspective of the program. There may not be a single number that fits all projects and programs to provide a meaningful expression of risk.

We have shown that all of the boxes in the NAVSEAINST 5100.12 chart are related by multiplicative factors. Therefore, if one box is assigned a value, the corresponding value for any other MRI can be known. For convenience, we choose to assign those elements on the Line of Mediocrity the value of 1, or

$$1E = 2D = 3C = 4A = 1 \qquad (3)$$

From there, all other indices can have a numerical value associated with the hazard.

To calculate the total risk of several independent hazards, one can simply add the numerical values together to obtain a single number. Table 3 shows a hypothetical example of how risk may be added.

Table 3 – Hypothetical Example of Risk Summation

Personnel shocked/electrocuted during maintenance	1E	1.000
Radiation hazard from active radar	3D	0.778
Hardware comes loose during a shock-inducing event	3D	0.778
Hardware not properly secured	3B	12.85
TOTAL CRI		15.406

Figure 5 depicts a graph showing the CRIs of the various risks on the NAVSEAINST 5100.12 chart.

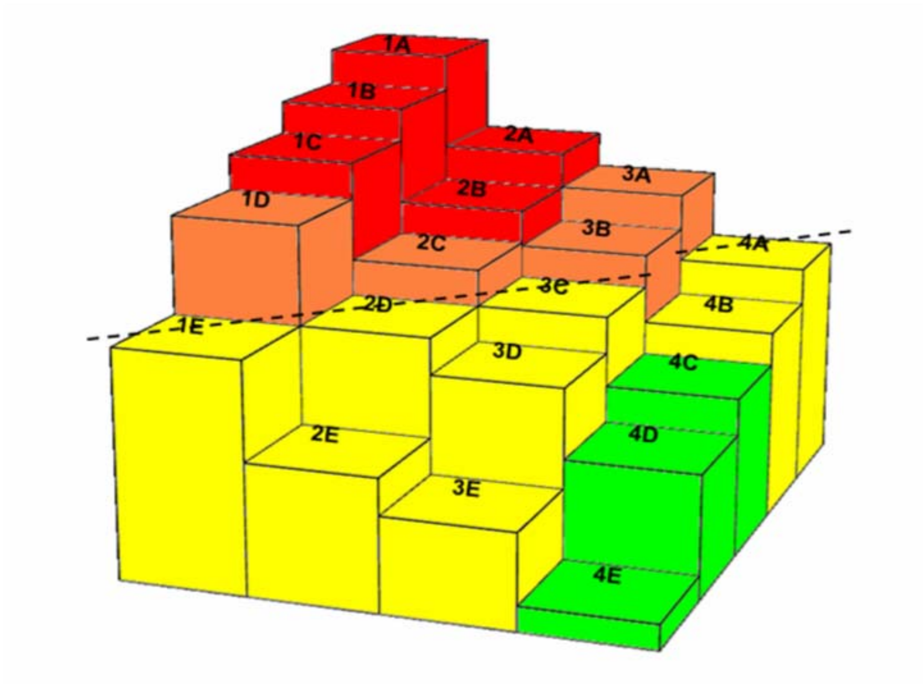

Figure 5 – Graphical Depiction of the Composite Risk Indices

The heights of the bars in Figure 5 are proportional to the logarithm of the CRI for each index. One can see that there are a few discrepancies:

1. 4D (low) has a higher CRI than 3E (medium)
2. 1D (serious) has a higher CRI than 2B (high)
3. 4C (low) has a higher CRI than either 2E and 3E (medium)
4. 3A (serious) has a slightly higher CRI than 2B (high)

These discrepancies could be resolved via a minor change to the NAVSEAINST 5100.12, as shown in Figure 6.

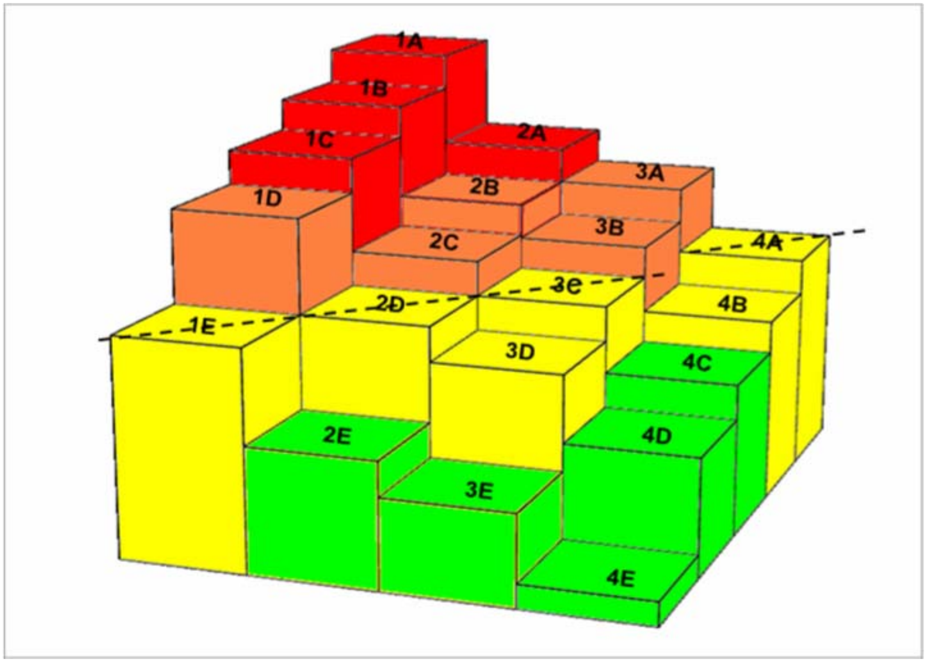

Figure 6 – Modified NAVSEAINST 5100.12 Chart

In Figure 6, 2B has been changed from high to serious, and 2E and 3E have been changed from medium to low. Given the changes, one could establish thresholds for high, serious, medium, and low based on the CRI.

Table 4 lists sample threshold values based on the CRI. Note that there is a variety of threshold values that could have been chosen. The particular method to determine the threshold CRI from low to medium was a number between the highest low (4C) and the lowest medium (3D). A similar approach was chosen to determine thresholds between medium and serious, and serious and high.

Table 4 – Sample CRI Thresholds

Risk	CRI
High	CRI > 20
Serious	20 > CRI > 2.5
Medium	2.5 > CRI > 0.135
Low	0.135 > CRI

One observation from Figure 6 compared to Figure 5 is the color bands appear to be more linear in Figure 6 than Figure 5. Current work on NAVSEAINST 5100.12B (currently in draft) suggests using even powers of 10 for probability and including more severities.

Risk can be quantified in this manner; however, it is important to note that:

1. Hazards should be independent of each other for optimal results. If they are related, the CRI will become increasingly less accurate. Instead, hazards identified to be related (e.g., operation of radars and working aloft) should be combined together and treated as a single hazard.
2. High precision numbers are required if there are a disparate distribution in the seriousness of the individual hazards.

3. The CRI should not be construed as an absolute measuring stick. It is only as good as the fidelity of the data that supports it. In general, specific probabilities of occurrence are not precisely known, and there is some subjectivity in the severity of the hazard.

References

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Biography

Mr. Banerjee is a System Safety Engineer at the Naval Surface Warfare Center in Dahlgren, VA. His expertise is in software safety.